The Bifurcation Diagram:

Major Results

Equation: $f_\lambda(x) = \lambda x(1-x)$

We made 4 bifurcation diagrams (the number of iterations is k):

a) $50 \leq k \leq 250$, 3000 values of λ

b) $50 \leq k \leq 250$, 30,000 values of λ

c) $50 \leq k \leq 250$, 300,000 values

d) $500 \leq k \leq 2500$, 3000 values of λ

1. In the period 1 window $1 \leq \lambda \leq 3.57$, there are infinitely many bifurcations. In graph c, about 8 bifurcations can be seen clearly. A bifurcation occurs when a fixed point slowly becomes a 2-cycle. Then the points of the bifurcation get farther and farther apart, resulting in the second bifurcation. This process continues until the chaos is reached.
2. Notice that there are smears near the bifurcation. The points next to the bifurcation don't have time to converge to a fixed points. Hence, given more iterations, like graph d, there is less smear.
3. Any window that is contained in the white stripe $3.828 \leq \lambda \leq 3.857$, must have period at least 3. In particular, they must be of the form 3×2^n, where n is the number of bifurcation that happened till the start of the window. During chaos, if every value λ of a window is in chaos, the period is extremely large, and may get larger if the interval of iterations is increased.
4. There are darker curves in some parts of the diagram, especially in a. These are where the graph is very dense, because the logistic map is not uniformly distributed.

The graph enters a state of chaos at $1+\sqrt{8} \approx 3.57$.

6. The bifurcation map has self-similar and fractal-like properties. For example, if you take the area above the top-most bifurcation, the others are a smaller version of the bigger one, similarly with the bottom-most bifurcation. A very similar thing happens with the white stripes in the chaos, and the black, dense, curves, in the chaos.

7. Let μ_n be the value of λ at the point where the map seperates into 2^n cycle. Then $\lim_{n\to\infty} \frac{\mu_n - \mu(n-1)}{\mu(n+1) - \mu(n)} = \delta$, or feigenbaum's constant, $\delta \approx 4.669201609102\,99$. This is true for <u>all</u> bifurcation maps, not just the logistic map.